> *restart*

> *# Exercice 1*

> $isolate\left(\mathrm{sqrt}(n+1)-\mathrm{sqrt}(n)=\frac{1}{2\cdot\mathrm{sqrt}(n+u)}, u\right)$

$$u=\frac{1}{\left(2\sqrt{n+1}-2\sqrt{n}\right)^2}-n \quad (1)$$

> $limit\left(\frac{1}{\left(2\sqrt{n+1}-2\sqrt{n}\right)^2}-n, n=\mathrm{infinity}\right)$

$$\frac{1}{2} \quad (2)$$

> *# Exercice 2*

> $rsolve(\{f(n+2)=f(n+1)+2\cdot f(n), f(0)=-4, f(1)=-2\}, f)$

$$-2\,(-1)^n-2\,2^n \quad (3)$$

> *# Exercice 3*

> $limit\left(product\left(2^{\frac{k}{2^k}}, k=1..n\right), n=\mathrm{infinity}\right)$

$$4 \quad (4)$$

> $limit\left(sum\left(\frac{1}{k^2}, k=1..n\right), n=\mathrm{infinity}\right)$

$$\frac{\pi^2}{6} \quad (5)$$

> $limit\left(int\left(\frac{1}{1+x^2}, x=0..n\right), n=\mathrm{infinity}\right)$

$$\frac{\pi}{2} \quad (6)$$

> *# Exercice 4*

> $rsolve(\{f(n+2)=f(n+1)+f(n), f(0)=1, f(1)=1\}, f)$

$$\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n+\left(\frac{1}{2}+\frac{\sqrt{5}}{10}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n \quad (7)$$

> $f:=n\to simplify\left(\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n+\left(\frac{1}{2}+\frac{\sqrt{5}}{10}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n\right):$

> $seq(f(n), n=1..20)$

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946 **(8)**

> *restart*

> $f:=x\to\mathrm{sqrt}(x-1); g:=(x,y)\to x^2-y^2$

$$f:=x\mapsto\sqrt{x-1}$$

$$g:=(x,y)\mapsto x^2-y^2 \quad (9)$$

> *# Exercice 5*

> $isolate\left(\mathrm{sqrt}(1+x)=1+\frac{x}{2\cdot \mathrm{sqrt}(1+x\cdot t)}, t\right)$

$$t=\frac{\frac{x^2}{4\left(-\sqrt{1+x}+1\right)^2}-1}{x} \quad \textbf{(10)}$$

> $limit\left(\frac{\frac{x^2}{4\left(-\sqrt{1+x}+1\right)^2}-1}{x}, x=0\right)$

$$\frac{1}{2} \quad \textbf{(11)}$$

> *# Exercice 6*

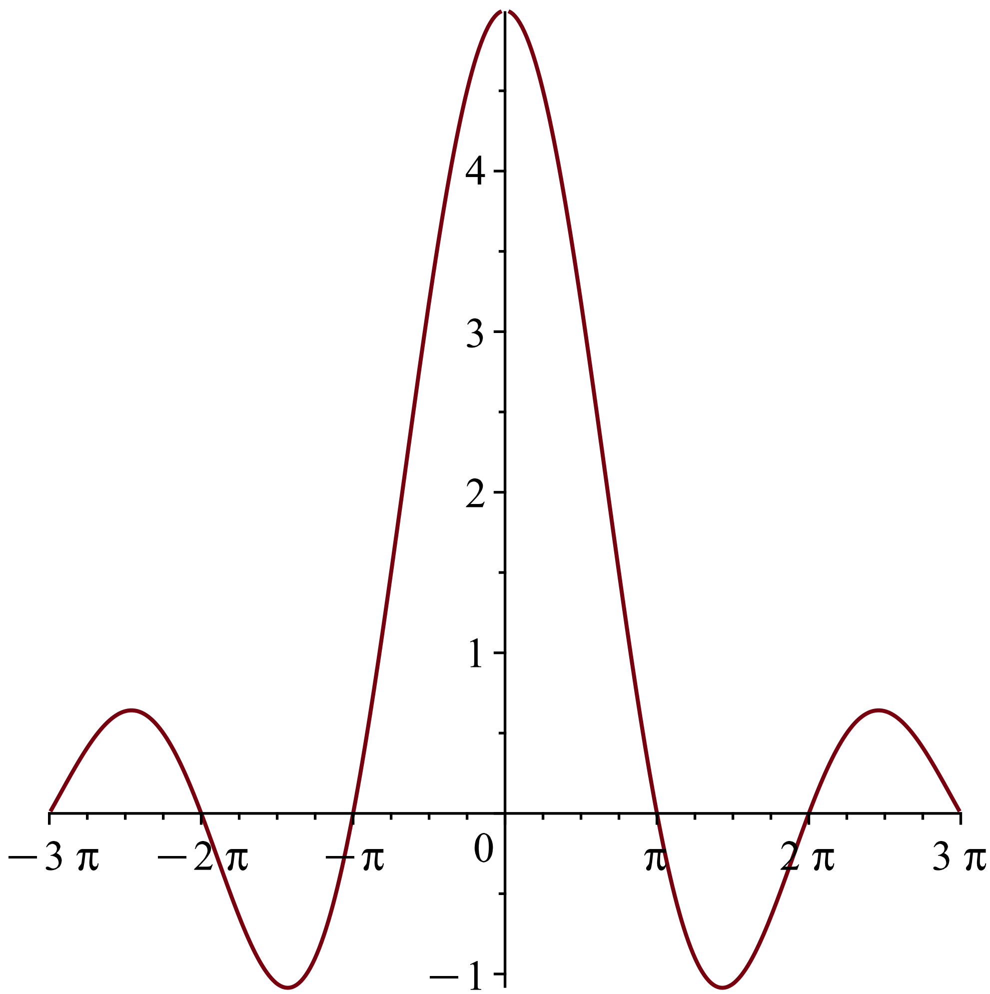
> $restart; limit\left(\frac{5\cdot \sin(x)}{x}, x=0\right)$

$$5 \quad \textbf{(12)}$$

> $f := x \rightarrow \textbf{if } x=0 \textbf{ then } 5 \textbf{ else } \frac{5\cdot \sin(x)}{x} \textbf{ fi}$

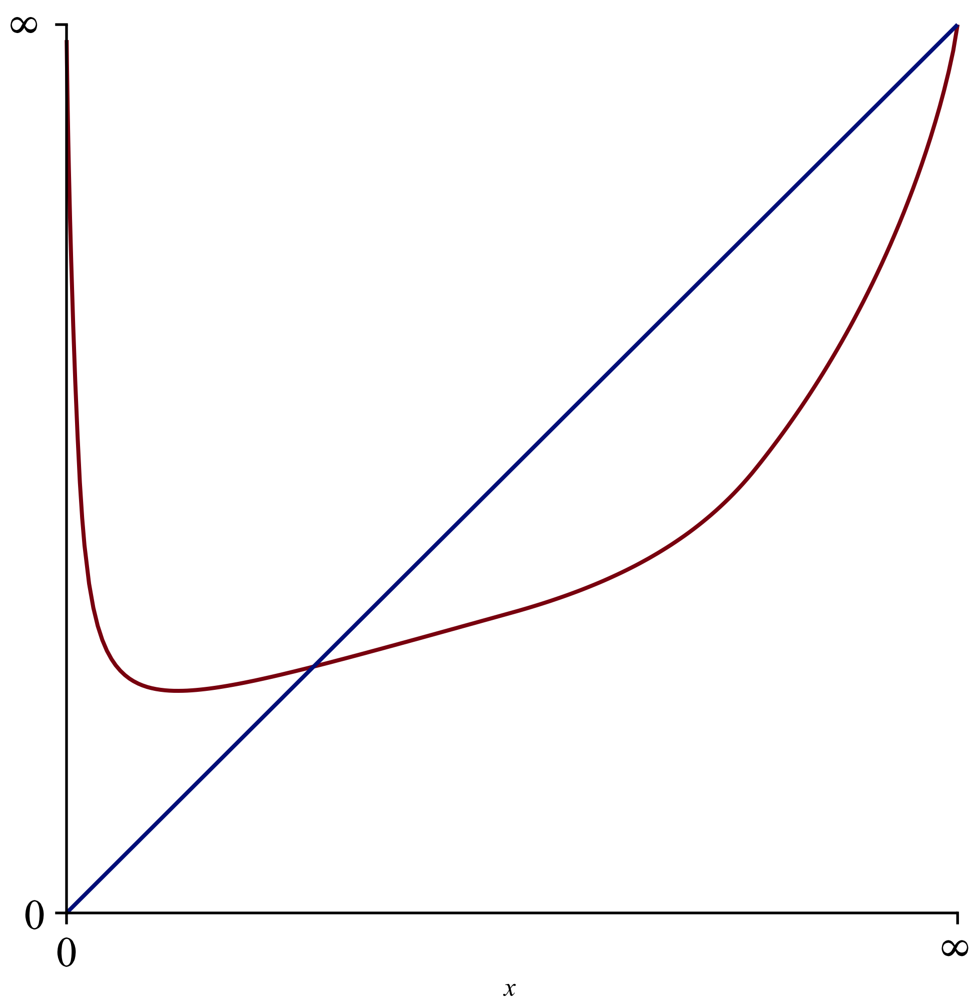
$$f := x \mapsto \textbf{if } x=0 \textbf{ then } 5 \textbf{ else } \frac{5\cdot \sin(x)}{x} \textbf{ end if} \quad \textbf{(13)}$$

> $plot(f, -3\cdot \mathrm{Pi}..3\cdot \mathrm{Pi})$



> *# Exercice 7*

> $plot\left(\left[\frac{x^2-1}{x\cdot\ln(x)}, x\right], x = 0\,..\text{infinity}\right)$



> $limit\left(\frac{x^2-1}{x\cdot\ln(x)}-x, x=\text{infinity}\right); limit\left(\frac{x^2-1}{x\cdot\ln(x)}-x, x=0\right); limit\left(\frac{x^2-1}{x\cdot\ln(x)}-x, x=1\right)$

$$-\infty$$

$$undefined$$

$$1 \tag{14}$$

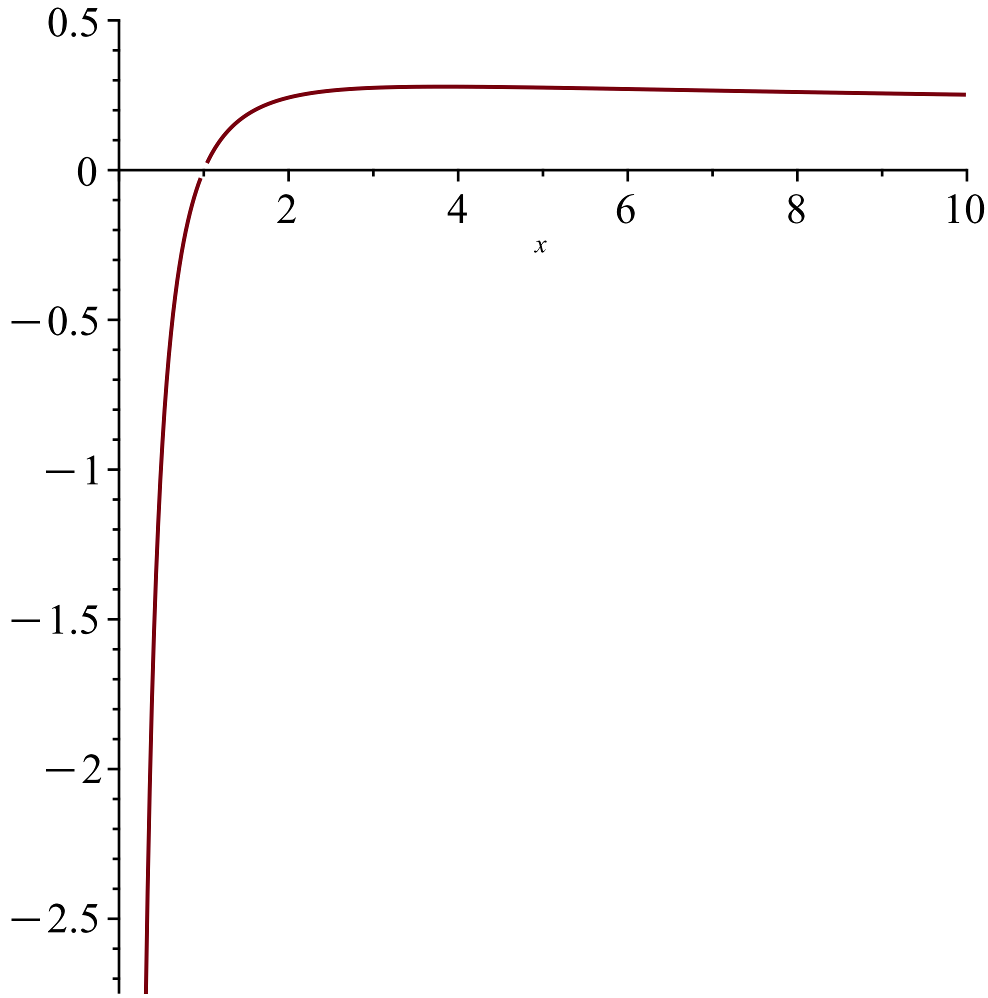
> $solve\left(diff\left(\frac{x^2-1}{x\cdot\ln(x)}, x\right)=0, x\right)$

$$e^{RootOf\left(\left(e^{\_Z}\right)^2 \_Z - \left(e^{\_Z}\right)^2 + \_Z + 1\right)} \tag{15}$$

> $evalf\left(e^{RootOf\left(\left(e^{\_Z}\right)^2 \_Z - \left(e^{\_Z}\right)^2 + \_Z + 1\right)}\right)$

$$1. \tag{16}$$

> $plot\left(diff\left(\frac{x^2-1}{x\cdot\ln(x)}, x\right)\right)$



> $limit\left(\frac{x^2-1}{x\cdot\ln(x)}-x, x=0, right\right); limit\left(\frac{x^2-1}{x\cdot\ln(x)}-x, x=0, left\right)$

$$\infty$$
$$-\infty \qquad (17)$$

> $limit\left(\frac{x^2-1}{x\cdot\ln(x)}, x=1\right); limit\left(\frac{x^2-1}{x\cdot\ln(x)}, x=0\right);\ limit\left(\frac{x^2-1}{x\cdot\ln(x)}, x=\text{infinity}\right)$

$$2$$
$$undefined$$
$$\infty \qquad (18)$$

> $limit\left(\frac{x^2-1}{x\cdot\ln(x)}, x=0, right\right), limit\left(\frac{x^2-1}{x\cdot\ln(x)}, x=0, left\right)$

$$\infty, -\infty \qquad (19)$$

> *# Bonus*

> $rsolve(\{v(n+3)=v(n+2)+v(n+1)+2\cdot v(n), v(0)=1, v(1)=0, v(2)=0\}, v)$

$$-\frac{4\,\mathrm{I}\left(\frac{9\,\mathrm{I}}{4}-\frac{\sqrt{3}}{4}\right)\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{21}-\frac{4\,\mathrm{I}\left(\frac{9\,\mathrm{I}}{4}+\frac{\sqrt{3}}{4}\right)\left(-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{21}+\frac{2^n}{7} \qquad (20)$$

$$> simplify\left(subs\left(n=100, \frac{2^n}{7} - \frac{4\,\mathrm{I}\left(\frac{9\,\mathrm{I}}{4}+\frac{\sqrt{3}}{4}\right)\left(-\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{21} - \frac{4\,\mathrm{I}\left(\frac{9\,\mathrm{I}}{4}-\frac{\sqrt{3}}{4}\right)\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{21}\right)\right)$$

$$181092942889747057356671886482 \qquad \textbf{(21)}$$

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